# INTRODUCTION TO DATA SCIENCE 

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Midterm Review - 10/27/2020
CMSC320
Tuesdays \& Thursdays
5:00pm - 6:15pm
(... or anytime on the Internet)


COMPUTER SCIENCE UNIVERSITY OF MARYLAND

## ANNOUNCEMENTS

Mini-Project \#2 was due late last week!

- Deliverable was an .ipynb file submitted to ELMS, but moving forward this will be .pdf / .html files, for TA grading ease
- Some folks had trouble getting the .pdf export to render figures - that's okay, if we run into an issue grading, we'll ping you
- In the future: can export to .html and then convert to .pdf Mini-Project \#3 will be released after the midterm!
- Due before Thanksgiving (TBD)


## PROJECT 1 GRADES ARE UPI

General comments:


People did really well!
We used a fairly strict rubric, but if you have a real bone to pick with your grade, please triage through TAs/office hours!

Comments for our sanity, moving forward:

- df. head ( n ) -- defaults to $\mathrm{n}=5$, use $\sim 10,20,50$ as needed
- Please label your pdf file something like <lastname>_<firstname>_project3.pdf
- E.g., dickerson_john_project3.pdf


## MIDTERM: STRUCTURE

50 points $=25 \%$ of the total grade
Rough breakdown (may change a little):
10 points:

- 10 True/False questions, 1 point each

10 points:

- 5 multiple choice questions, 2 points each

30 points:

- 10 short answer questions, 3 points each

Compared to the CMSC320 midterm I posted from an earlier semester, this midterm is more qualitative.

## QUICK MIDTERM REVIEW

As discussed in previous lectures and on Piazza, the midterm can cover:

- Up to and including last Thursday's lecture (10/22)
- Quizzes that were due on or before today
- Stuff that you should know from doing P1 and P2

Everything is online: https://cmsc320.github.io/
I know this is a lot of material.

- Rule of thumb: open up a slide deck
- Do you feel "comfortable" with the material?
- Test will be more qualitative than prior 1xx, 2xx, 3xx tests


## QUICK MIDTERM REVIEW



## DATA COLLECTION (DC) \& DATA PROCESSING (DP)

We talked about:

- Scraping data
- RESTful APIs
- Structured data formats (JSON, XML, etc)
- Regexes

Data manipulation via Numpy Stack (Numpy, Pandas, etc)

- Indexing, slicing, groups, joins, aggregate queries, etc

Tidy data + melting
Version control (just know how this works qualitatively) RDMS, a little bit of SQL
Entity resolution \& other data integration issues
Storing stuff as a graph, and manipulating it

## DC: HTTP REQUESTS

https://www.google.com/?q=cmsc320\&tbs=qdr:m
Google
??????????

## HTTP GET Request:

GET /?q=cmsc320\&tbs=qdr:m HTTP/1.1
Host: www.google.com
User-Agent: Mozilla/5.0 (X11; Linux x86_64; rv:10.0.1) Gecko/20100101 Firefox/10.0.1

```
params = { "q": "cmsc320", "tbs": "qdr:m" }
r = requests.get( "https://www.google.com",
    params = params )
```


## DC: RESTFUL APIS

This class will just query web APIs, but full web APIs typically allow more.

Representational State Transfer (RESTful) APIs:

- GET: perform query, return data
- POST: create a new entry or object
- PUT: update an existing entry or object
- DELETE: delete an existing entry or object

Can be more intricate, but verbs ("put") align with actions


## DC: PANDAS: SERIES

index values


- Subclass of numpy.ndarray
- Data: any type
- Index labels need not be ordered
- Duplicates possible but result in reduced functionality


## DC: PANDAS: DATAFRAME



- Each column can have a different type
- Row and Column index
- Mutable size: insert and delete columns
- Note the use of word "index" for what we called "key"
- Relational databases use "index" to mean something else
- Non-unique index values allowed
- May raise an exception for some operations


## DC: STORING A <br> GRAPH

Three main ways to represent a graph in memory:

- Adjacency lists
- Adjacency dictionaries
- Adjacency matrix

The storage decision should be made based on the expected use case of your graph:

- Static analysis only?
- Frequent updates to the structure?
- Frequent updates to semantic information?


## DC: ADJACENCY LISTS

For each vertex, store an array of the vertices it connects to


| Vertex | Neighbors |
| :--- | :--- |
| $A$ | $[C]$ |
| $B$ | $[C, D]$ |
| $C$ | $[A]$ |
| $D$ | [] |

Pros: ????????

- Iterate over all outgoing edges; easy to add an edge

Cons: ????????

- Checking for the existence of an edge is $\mathrm{O}(|\mathrm{V}|)$, deleting is hard


## DC: ADJACENCY DICTIONARIES

For each vertex, store a dictionary of vertices it connects to


| Vertex | Neighbors |
| :--- | :--- |
| A | $\{C: 1.0\}$ |
| $B$ | $\{C: 1.0$, D: 1.0$\}$ |
| C | $\{A: 1.0\}$ |
| $D$ | $\}$ |

Pros: ?????????

- $\mathrm{O}(1)$ to add, remove, query edges

Cons: ?????????

- Overhead (memory, caching, etc)


## DC: ADJACENCY MATRIX

Store the connectivity of the graph in a matrix


Cons: ?????????


- $\mathrm{O}\left(|\mathrm{V}|^{2}\right)$ space regardless of the number of edges

Almost always stored as a sparse matrix

## DP: SELECT/SLICING

Select only some of the rows, or some of the columns, or a combination

| ID | age | wgt kg | hgt cm | Only columns |  | 12.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 12.2 | 42.3 | 145.1 | and Age |  | 11.0 |
| 2 | 11.0 | 40.8 | 143.8 |  |  | 15.6 |
| 3 | 15.6 | 65.3 | 165.3 |  |  | 35.1 |
| 4 | 35.1 | 84.2 | 185.8 |  |  |  |
| Only rows with wgt > 41 |  |  |  | Bo |  | age |
| ID | age | wgt_kg | hgt_cm |  |  | 12.2 |
| 1 | 12.2 | 42.3 | 145.1 |  |  | 15.6 |
| 3 | 15.6 | 65.3 | 165.3 |  |  |  |
| 4 | 35.1 | 84.2 | 185.8 |  |  |  |

## DP: AGGREGATE/REDUCE

Combine values across a column into a single value


## DP: MAP

Apply a function to every row, possibly creating more or fewer columns

| ID | Address |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | College Park, MD, 20742 |
| 2 | Washington, DC, 20001 |
| 3 | Silver Spring, MD 20901 |$\longrightarrow$| ID | City | State | Zipcode |
| :--- | :--- | :--- | :--- |
| 1 | College <br> Park | MD | 20742 |
| 2 | Washington | DC | 20001 |
| 3 | Silver <br> Spring | MD | 20901 |

Variations that allow one row to generate multiple rows in the output (sometimes called "flatmap")

## DP: GROUP BY

Group tuples together by column/dimension

| ID | A | B | C |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |

By 'A'

| ID | B | C |
| :--- | :--- | :--- |
| 1 | 3 | 6.6 |
| 3 | 4 | 3.1 |
| 4 | 3 | 8.0 |
| 7 | 4 | 2.3 |
| 8 | 3 | 8.0 |


| A $=$ bar |  |  |
| :---: | :---: | :---: |
| ID | B | C |
| 2 | 2 | 4.7 |
| 5 | 1 | 1.2 |
| 6 | 2 | 2.5 |

$$
B=1
$$

## DP: GROUP BY

Group tuples together by column/dimension

| ID | A | B | C |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |


| ID | A | C |
| :--- | :--- | :--- |
| 5 | bar | 1.2 |


| ID | A | C |
| :--- | :--- | :--- |
| 2 | bar | 4.7 |
| 6 | bar | 2.5 |

By 'B'

| ID | A | C |
| :--- | :--- | :--- |
| 1 | foo | 6.6 |
| 4 | foo | 8.0 |
| 8 | foo | 8.0 |
| B $=4$ |  |  |
| ID | A | C |
| 3 | foo | 3.1 |
| 7 | foo | 2.3 |

$$
A=\text { bar, } B=1
$$

## DP: GROUP BY

| ID | C |
| :--- | :--- |
| 5 | 1.2 |

Group tuples together by column/dimension

| ID | A | B | C |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |

$\mathrm{A}=\mathrm{bar}, \mathrm{B}=2$

| ID | C |
| :--- | :--- |
| 2 | 4.7 |
| 6 | 2.5 |

$A=$ foo, $B=3$
By 'A', 'B'

| ID | C |
| :--- | :--- |
| 1 | 6.6 |
| 4 | 8.0 |
| 8 | 8.0 |

$A=$ foo, $B=4$

| ID | C |
| :--- | :--- |
| 3 | 3.1 |
| 7 | 2.3 |

## DP: GROUP BY AGGREGATE

Compute one aggregate Per group

| ID | A | B | C |
| :---: | :---: | :---: | :---: |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |


| ID | A | C |
| :--- | :--- | :--- |
| 2 | bar | 4.7 |
| 6 | bar | 2.5 |

$B=3$

| ID | A | c |  |
| :---: | :---: | :---: | :---: |
| 1 | foo | 6.6 | Sum (C) |
|  |  |  | 22.6 |
| 4 | foo | 8.0 |  |
| 8 | foo | 8.0 | $B=4$ |
| $\mathrm{B}=4$ |  |  | Sum (C) |
| ID | A | C | 5.4 |
| 3 | foo | 3.1 |  |
| 7 | foo | 2.3 |  |

## DP: GROUP B AGGREGATE

Final result usually seen

|  | tabl |  |  |  | $B=2$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ID | A | B | C |  | Sum (C) |  |  |
| 1 | foo | 3 | 6.6 |  | 7.2 |  | SUM(C) |
| 2 | bar | 2 | 4.7 |  |  |  | 1.2 |
| 3 | foo | 4 | 31 |  | $\mathrm{B}=3$ |  | 7.2 |
| 3 | foo |  | 8.1 | Group by 'B' |  |  | 22.6 |
| 4 | foo | 3 | 8.0 | Sum on C | Sum (C) |  | 5.4 |
| 5 | bar | 1 | 1.2 |  | 22.6 |  |  |
| 6 | bar | 2 | 2.5 |  |  |  |  |
| 7 | foo | 4 | 2.3 |  |  |  |  |
| 8 | foo | 3 | 8.0 |  | Sum (C) |  |  |
|  |  |  |  |  | 5.4 |  |  |

## DP: UNION/INTERSECTION/DIFFERENCE

Set operations - only if the two tables have identical attributes/columns

| ID | A | B | C |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |


| ID | A | B | C |
| :--- | :--- | :--- | :--- |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |

## Similarly Intersection and Set Difference manipulate tables as Sets

| ID | A | $\mathbf{B}$ | C |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 6.6 |
| 2 | bar | 2 | 4.7 |
| 3 | foo | 4 | 3.1 |
| 4 | foo | 3 | 8.0 |
| 5 | bar | 1 | 1.2 |
| 6 | bar | 2 | 2.5 |
| 7 | foo | 4 | 2.3 |
| 8 | foo | 3 | 8.0 |

IDs may be treated in different ways, resulting in somewhat different behaviors

## DP: MERGE OR JOIN

Combine rows/tuples across two tables if they have the same key

| ID | A | $\mathbf{B}$ |
| :--- | :--- | :--- |
| 1 | foo | 3 |
| 2 | bar | 2 |
| 3 | foo | 4 |
| 4 | foo | 3 |


| $\mathbf{I D}$ | $\mathbf{C}$ |
| :--- | :--- |
| 1 | 1.2 |
| 2 | 2.5 |
| 3 | 2.3 |
| 5 | 8.0 |$\longrightarrow$| $\mathbf{I D}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ |
| :--- | :--- | :--- | :--- |
| 1 | foo | 3 | 1.2 |
| 2 | bar | 2 | 2.5 |
| 3 | foo | 4 | 2.3 |

What about IDs not present in both tables?
Often need to keep them around
Can "pad" with NaN

## DP: MERGE OR JOIN

Combine rows/tuples across two tables if they have the same key
Outer joins can be used to "pad" IDs that don't appear in both tables
Three variants: LEFT, RIGHT, FULL
SQL Terminology -- Pandas has these operations as well

| ID | A | B |
| :--- | :--- | :--- |
| 1 | foo | 3 |
| 2 | bar | 2 |
| 3 | foo | 4 |
| 4 | foo | 3 |


| D | ID | C | D | A | B | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1.2 | 1 | foo | 3 | 1.2 |
|  | 2 | 2.5 | 2 | bar | 2 | 2.5 |
|  | 3 | 2.3 | 3 | foo | 4 | 2.3 |
|  | 5 | 8.0 | 4 | foo | 3 | NaN |
|  |  |  | 5 | NaN | NaN | 8.0 |

# DP: GOOGLE IMAGE SEARCH ONE SLIDE SQL JOIN VISUAL 



FULL JOIN



## DC/DP: HOW A RELATIONAL DB FITS INTO YOUR WORKFLOW

Persists!


SQLite CLI \& GUI Frontend

## DP: ADDITIONAL STUFF

Data integration

- Extraction, schema alignment \& mapping, querying over multiple schema / global schema
Data quality issues
- Single- vs multi-source quality issues

Data cleaning

- Outlier detection, constraint-based cleaning


## Entity resolution (~part of data cleaning)

- Deduplication, record linkage, reference matching
- Fuzzy matching, etc.


## EDA \& VIZ

Missing data

- MCAR
- MAR
- MNAR
- Single \& multiple imputation

Analysis

- Basic linear regression
- Summary statistics / robust statistics
- Variance, stdev, covariance, Pearson's correlation coefficient
- Hypothesis testing
- Bayes' rule.


## EDA: MISSING DATA

Missing data is information that we want to know, but don't
It can come in many forms, e.g.:

- People not answering questions on surveys
- Inaccurate recordings of the height of plants that need to be discarded
- Canceled runs in a driving experiment due to rain

Could also consider missing columns (no collection at all) to be missing data ...

## EDA: COMPLETE CASE ANALYSIS

Delete all tuples with any missing values at all, so you are left only with observations with all variables observed

```
# Clean out rows with nil values
df = df.dropna()
```

Default behavior for libraries for analysis (e.g., regression)

- We'll talk about this much more during the Stats/ML lectures

This is the simplest way to handle missing data. In some cases, will work fine; in others, ?????????????:

- Loss of sample will lead to variance larger than reflected by the size of your data
- May bias your sample


## EDA: YOUR SAMPLE

| Hair Color | $>6 f t$ | Grade |
| :---: | :---: | :---: |
| Red | Y | A |
| Brown | N | A |
| Black | N | B |
| Black | Y | A |
| Brown | Y |  |
| Brown | Y |  |
| Brown | N |  |
| Black | Y | B |
| Black | Y | B |
| Brown | N | A |
| Black | N |  |
| Brown | N | C |
| Red | Y |  |
| Red | N | A |
| Brown | Y | A |
| Black | Y | A |

## Summary:

- 7 students received As
- 3 students received Bs
- 1 student received a C

Nobody is failing!

- But 5 students did not reveal their grade ...


## EDA: WHAT INFLUENCES A DATA POINT'S PRESENCE?

Same dataset, but the values are replaced with a " 0 " if the data point is observed and " 1 " if it is not

Question: for any one of these data points, what is the probability that the point is equal to " 1 " ...?

What type of missing-ness do the grades exhibit?

| Hair Color | $>6 \mathbf{f t}$ | Grade |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

## EDA: MCAR: MISSING COMPLETELY AT RANDOM

If this probability is not dependent on any of the data, observed or unobserved, then the data is Missing Completely at Random (MCAR)

Suppose that $X$ is the observed data and $Y$ is the unobserved data. Call our "missing matrix" $R$

Then, if the data are MCAR, $P(R \mid X, Y)=$ ??????????

$$
P(R \mid X, Y)=P(R)
$$

Probability of those rows missing is independent of anything.

## EDA: MAR: MISSING AT

## RANDOM

Missing at Random (MAR): probability of missing data is dependent on the observed data but not the unobserved data Suppose that $X$ is the observed data and $Y$ is the unobserved data. Call our "missing matrix" $R$
Then, if the data are MAR, $\mathrm{P}(\mathrm{R} \mid \mathrm{X}, \mathrm{Y})=$ ??????????

$$
P(R \mid X, Y)=P(R \mid X)
$$

Not exactly random (in the vernacular sense).

- There is a probabilistic mechanism that is associated with whether the data is missing
- Mechanism takes the observed data as input


## EDA: MNAR: MISSING NOT AT RANDOM

MNAR: missing-ness has something to do with the missing data itself

Examples: ??????????

- Do you binge drink? Do you have a trust fund? Do you use illegal drugs? What is your sexuality? Are you depressed?
Said to be "non-ignorable":
- Missing data mechanism must be considered as you deal with the missing data
- Must include model for why the data are missing, and best guesses as to what the data might be


## EDA: BACK TO IRIBE

Is the the missing data:

- MCAR;
- MAR; or
- MNAR?
???????????


| Hair Color | $>6 f t$ | Grade |
| :---: | :---: | :---: |
| Red | Y | A |
| Brown | N | A |
| Black | N | B |
| Black | Y | A |
| Brown | Y |  |
| Brown | Y |  |
| Brown | N |  |
| Black | Y | B |
| Black | Y | B |
| Brown | N | A |
| Black | N |  |
| Brown | N | C |
| Red | Y |  |
| Red | N | A |
| Brown | Y | A |
| Black | Y | A |

## EDA: ADD A VARIABLE

## Bring in the GPA:

Does this change anything?

| Hair Color | GPA | Gender | Grade |
| :---: | :---: | :---: | :---: |
| Red | 3.4 | M | A |
| Brown | 3.6 | F | A |
| Black | 3.7 | F | B |
| Black | 3.9 | M | A |
| Brown | 2.5 | M |  |
| Brown | 3.2 | M |  |
| Brown | 3.0 | F |  |
| Black | 2.9 | M | B |
| Black | 3.3 | M | B |
| Brown | 4.0 | F | A |
| Black | 3.65 | F |  |
| Brown | 3.4 | F | C |
| Red | 2.2 | M |  |
| Red | 3.8 | F | A |
| Brown | 3.8 | M | A |
| Black | 3.67 | M | A |

## EDA: MULTIPLE IMPUTATION PROCESS



## ANALYSIS: IMPORTANCE OF VERTICES

Not all vertices are equally important
Centrality Analysis:

- Find out the most important node(s) in one network
- Used as a feature in classification, for visualization, etc ...

Commonly-used Measures

- Degree Centrality
- Closeness Centrality
- Betweenness Centrality
- Eigenvector Centrality


## ANALYSIS: DEGREE CENTRALITY

The importance of a vertex is determined by the number of vertices adjacent to it

- The larger the degree, the more important the vertex is
- Only a small number of vertex have high degrees in many reallife networks
Degree Centrality: $\quad C_{D}\left(v_{i}\right)=d_{i}=\sum_{j} A_{i j}$
Normalized Degree Centrality: $\quad C_{D}^{\prime}\left(v_{i}\right)=d_{i} /(n-1)$


For vertex 1 , degree centrality is 3 ;
Normalized degree centrality is $3 /(9-1)=3 / 8$.

## ANALYSIS: BETWEENNESS CENTRALITY



| Table 2.2: |  |  |  |
| :---: | :---: | :---: | :---: |
| $\sigma_{s t}(4) / \sigma_{s t}$ |  |  |  |
| $t=5$ | $1 / 1$ | $2 / 2$ | $1 / 1$ |
| $t=6$ | $1 / 1$ | $2 / 2$ | $1 / 1$ |
| $t=7$ | $2 / 2$ | $4 / 4$ | $2 / 2$ |
| $t=8$ | $2 / 2$ | $4 / 4$ | $2 / 2$ |
| $t=9$ | $2 / 2$ | $4 / 4$ | $2 / 2$ |

$\sigma_{s t}$ : The number of shortest paths between s and t
$\sigma_{s t}\left(v_{i}\right)$ : The number of shortest paths between s and t that pass $\mathrm{v}_{\mathrm{i}}$

$$
C_{B}\left(v_{i}\right)=\sum_{v_{s} \neq v_{i} \neq v_{t} \in V, s<t} \frac{\sigma_{s t}\left(v_{i}\right)}{\sigma_{s t}}
$$

What is the betweenness centrality for node 4 ?????????

## ANALYSIS: TERM FREQUENCY

Term frequency: the number of times a term appears in a specific document

- $\mathrm{tf}_{\mathrm{ij}}$ : frequency of word $j$ in document $i$

This can be the raw count (like in the BOW in the last slide):

- $\mathrm{tf}_{i j} \in\{0,1\}$ if word $j$ appears or doesn't appear in doc $i$
- $\log \left(1+\mathrm{tf}_{\mathrm{ij}}\right)$ - reduce the effect of outliers
- $\mathrm{tf}_{i j} /$ max $_{\mathrm{j}} \mathrm{tf}_{i j}$ - normalize by document i's most frequent word

What can we do with this?

- Use as features to learn a classifier $w \rightarrow y$...!


## ANALYSIS: INVERSE DOCUMENT FREQUENCY

Recall:

- $\mathrm{tt}_{i j}$ : frequency of word $j$ in document $i$

Any issues with this ??????????

- Term frequency gets overloaded by common words

Inverse Document Frequency (IDF): weight individual words negatively by how frequently they appear in the corpus:

$$
\operatorname{idf}_{j}=\log \left(\frac{\# \text { documents }}{\# \text { documents with word } j}\right)
$$

IDF is just defined for a word j , not word/document pair $\mathrm{j}, \mathrm{i}$

## ANALYSIS: TF-IDF

How do we use the IDF weights?
Term frequency inverse document frequency (TF-IDF):

- TF-IDF score: $\mathrm{tf}_{\mathrm{ij}} \times \mathrm{idf}_{j}$


This ends up working better than raw scores for classification and for computing similarity between documents.

## ANALYSIS: SIMILARITY BETWEEN DOCUMENTS

Given two documents $x$ and $y$, represented by their TF-IDF vectors (or any vectors), the cosine similarity is:

$$
\operatorname{similarity}(\mathbf{x}, \mathbf{y})=\frac{\mathbf{x}^{\top} \mathbf{y}}{|\mathbf{x}| \times|\mathbf{y}|}
$$

Formally, it measures the cosine of the angle between two vectors $x$ and $y$ :

- $\cos \left(0^{\circ}\right)=1, \cos \left(90^{\circ}\right)=0 \quad$ ??????????

Similar documents have high cosine similarity; dissimilar documents have low cosine similarity.


